

# CONTENTS IN DETAIL

<b>ACKNOWLEDGMENTS</b>	<b>xiii</b>
------------------------	-------------

<b>INTRODUCTION</b>	<b>xv</b>
---------------------	-----------

Why Scratch? .....	xv
Who This Book Is For .....	xvi
What's in This Book .....	xvi
How to Read This Book .....	xvii
Online Resources .....	xviii
My Inspiration .....	xviii

<b>1</b>	
<b>WHAT COMPUTERS THINK ABOUT NUMBERS</b>	<b>1</b>

What Are Numbers, Anyway? .....	2
Base 10? Base 2? You Pick! .....	2
<b>Project 1: What's 77 in Binary?</b> .....	<b>3</b>
<b>Project 2: What's 1001101 in Decimal?</b> .....	<b>6</b>
How Computers Represent Numbers .....	10
The Point of Floating Point .....	11
Double the Precision, Double the Fun .....	12
<b>Project 3: <math>2^{53} + 1 = ?</math></b> .....	<b>13</b>
<b>Project 4: A Million-Digit Number?</b> .....	<b>15</b>
Conclusion .....	19

<b>2</b>	
<b>EXPLORING DIVISIBILITY AND PRIMES</b>	<b>21</b>

The Divisibility Factor .....	21
Modular Arithmetic .....	22
<b>Project 5: A Trick for Checking Your Math</b> .....	<b>23</b>
Prime Numbers .....	27
<b>Project 6: Is It Prime?</b> .....	<b>28</b>
<b>Project 7: The Sieve of Eratosthenes</b> .....	<b>31</b>
Nothing Common About Common Divisors .....	36
<b>Project 8: Greatest Common Divisors the Slow Way</b> .....	<b>36</b>
<b>Project 9: Greatest Common Divisors the Fast Way</b> .....	<b>37</b>
Conclusion .....	40

<b>3</b>		
	<b>SPLITTING NUMBERS WITH PRIME FACTORIZATION</b>	<b>41</b>
	The Fundamental Theorem of Arithmetic .....	41
	<b>Project 10: Is It a Prime Factor?</b> .....	<b>42</b>
	Fun with Divisors .....	45
	<b>Project 11: Tau Many Divisors?</b> .....	<b>47</b>
	<b>Project 12: Summing Up to Sigma</b> .....	<b>47</b>
	How Prime Factorization Helps Find GCDs .....	50
	Contacting Aliens with Biprimes .....	52
	<b>Project 13: Fermat's Factorization Feat</b> .....	<b>54</b>
	Conclusion .....	57

<b>4</b>		
	<b>FINDING PATTERNS IN SEQUENCES</b>	<b>59</b>
	What Are Sequences? .....	59
	Finding the Next Value in a Sequence .....	60
	Making Sequences in Scratch .....	61
	<b>Project 14: Fibonacci's Rabbits</b> .....	<b>61</b>
	<b>Project 15: The Golden Ratio</b> .....	<b>62</b>
	Figurate Numbers .....	66
	<b>Project 16: Square, Triangular, and Pentagonal Numbers?</b> .....	<b>67</b>
	Predicting Values in a Sequence .....	72
	<b>Project 17: Difference Tables Make All the Difference</b> .....	<b>72</b>
	Conclusion .....	78

<b>5</b>		
	<b>FROM SEQUENCES TO ARRAYS</b>	<b>79</b>
	Pascal's Triangle .....	80
	Working with Binomials .....	80
	Making Subsets from Sets .....	82
	<b>Project 18: Pick a Number from Pascal's Triangle</b> .....	<b>83</b>
	Pascal's Recurrence .....	86
	<b>Project 19: Pascal's Triangle, Row by Row</b> .....	<b>87</b>
	<b>Project 20: Drawing Pascal's Triangle</b> .....	<b>88</b>
	Operation Tables Have All the Answers .....	92
	<b>Project 21: Infinite Operation Tables with Modular Arithmetic</b> .....	<b>93</b>
	Conclusion .....	99

<b>6</b>	
<b>MAKING CODES, AND CRACKING THEM TOO</b>	<b>101</b>
Caesar's Shifty Cipher .....	101
<b>Project 22: Encryption by a Caesar Shift</b> .....	<b>103</b>
<b>Project 23: Cracking the Caesar Cipher</b> .....	<b>108</b>
More Substitution Ciphers .....	110
Encryption by Modular Multiplication .....	111
Decryption by Modular Multiplication .....	112
<b>Project 24: The Modular Inverse Is the Key</b> .....	<b>113</b>
More Encryption Options with Linear Transformations .....	116
<b>Project 25: Encryption by a Linear Transformation</b> .....	<b>117</b>
Unbreakable One-Time Pad Ciphers .....	120
<b>Project 26: Frequency Analysis for Cracking Codes</b> .....	<b>121</b>
<b>Project 27: Encryption with a One-Time Pad</b> .....	<b>124</b>
Conclusion .....	128

<b>7</b>	
<b>EXPERIMENTS IN COUNTING</b>	<b>129</b>
What Are Counting Problems? .....	129
Climbing Mountains with Catalan Numbers .....	130
<b>Project 28: Navigating Catalan Paths</b> .....	<b>132</b>
Breaking Down Numbers with Addition .....	138
Compositions: Order Does Matter .....	138
Partitions: Order Doesn't Matter .....	138
<b>Project 29: A Partition Expedition</b> .....	<b>139</b>
Conclusion .....	150

<b>8</b>	
<b>THREE HELPINGS OF PI</b>	<b>151</b>
How Archimedes Calculated Pi .....	152
<b>Project 30: Archimedes's Recurrence</b> .....	<b>153</b>
Estimating Pi from the Area of a Circle .....	156
<b>Project 31: Using the Lattice Point Tally</b> .....	<b>157</b>
Approximating Pi with Relative Primes .....	160
<b>Project 32: Using Only Visible Lattice Points</b> .....	<b>161</b>
Conclusion .....	164

<b>9</b>	
<b>WHAT NEXT?</b>	<b>165</b>
Learning Other Languages .....	165
Finding More Problems .....	167
<b>Project 33: Hacking Project Euler Problem 1 .....</b>	<b>167</b>
Beyond Project Euler .....	171
More Scratch Projects to Explore.....	171
Conclusion .....	173
<b>APPENDIX: PROGRAMMING CHALLENGE HINTS</b>	<b>175</b>
<b>INDEX</b>	<b>215</b>